ADER scheme for incompressible Navier-Stokes equations on Overset grids with a compact transmission condition

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ABSTRACT

In the present work, a second order Finite Volume (FV) approach is proposed for the solution of Navier-Stokes equations for incompressible fluids over an overset configuration of meshes. In particular, an overset grid (or chimera mesh) [1] is a composition of meshes allowing to overcome the problem of the geometrical adaptation of the computational domain with a unique block of mesh. These meshes overlap each other and can move and deform during the simulation.

The Navier-Stokes equations are solved through a projection method (*Chorin-Temam*). For this reason, two different solvers are presented for the nonlinear unsetady Advection-Diffusion (AD) problem (i.e. the *prediction* of the intermediate velocity field) and the steady diffusive problem (namely the Poisson problem for the pressure in the *projection* phase).

The AD problem is numerically solved by properly adapting the prediction-correction ADER method on chimera meshes [2]. First, the motion equation for the overlapping mesh is solved in order to evolve the overset grid between two consecutive discrete times t^n and t^{n+1}. Then, through an isogeometric approach, a *local predictor solution* of AD problem is found by knowledge of the solution at time t^n. Successively, by treating the time variable as a particular spatial direction, the AD problem is written in a hyperbolic form. Finally, in a space-time FV frame and through a Local Lax-Friederichs flux along the boundary of space-time cells by exploiting the local predictor solution, the *corrected* velocity field at time t^{n+1} is recovered.

For the Poisson equation, the FV approach involves the approximation of the normal pressure gradient along the edges of each mesh cell. In particular, if the cell is not at the overlapping zone, the normal gradients along the edges are approximated by properly combining the gradients along two consecutive cells and two consecutive vertexes; otherwise the normal gradients of the cell is built as a weighted expansion of pressures in the neighbouring cells by minimising a proper convex functional [3].

References

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